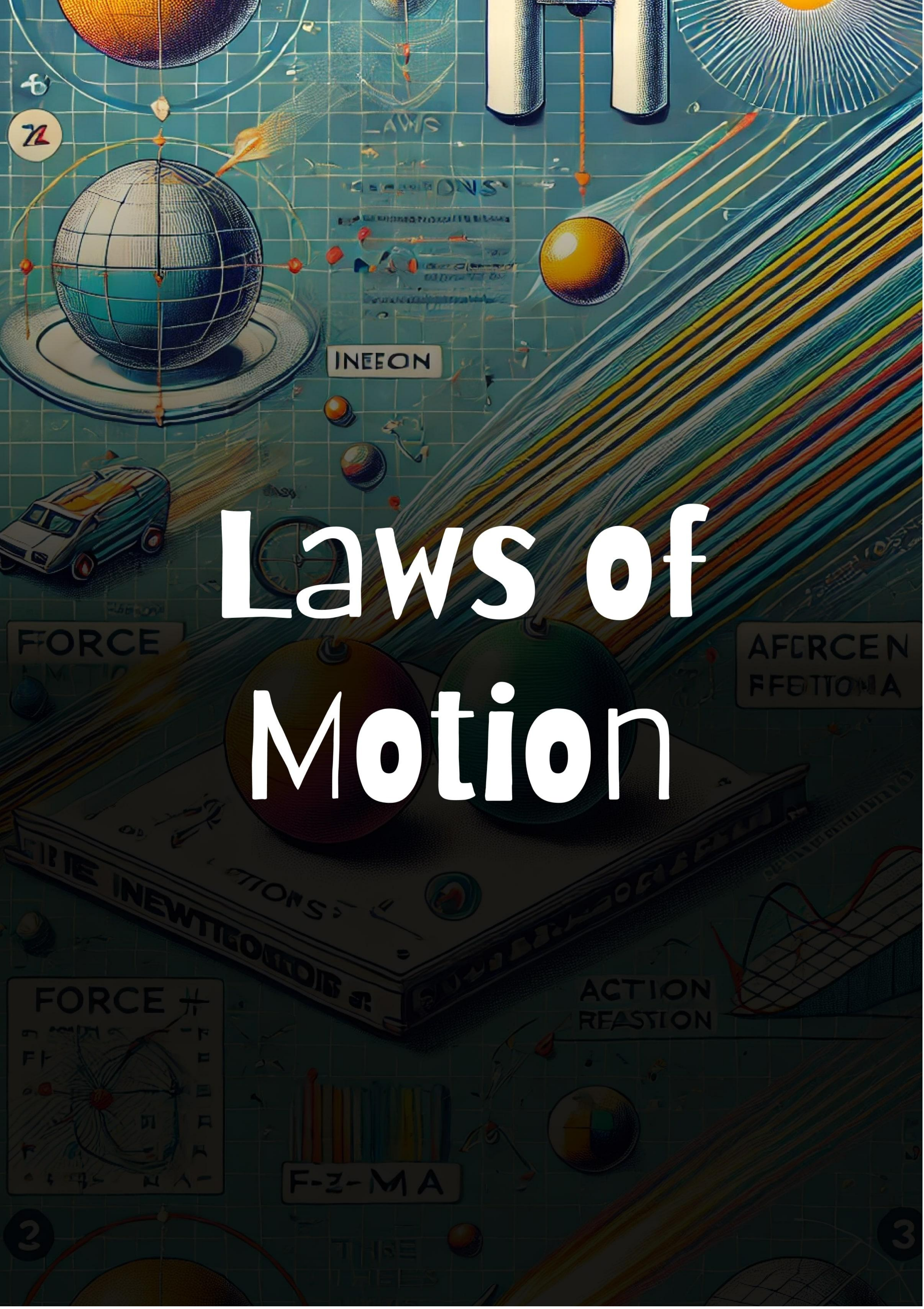


Laws of Motion

laws of motion

FORCE

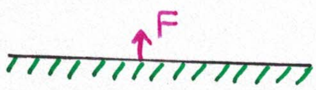
A force is a push, pull or a shear or a combination there off.

PURE PUSH

A force directed towards the surface and perpendicular to the surface.

PURE PULL

A force diverted away from the surface and perpendicular to the surface.



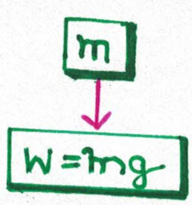
SHEAR

A force directed parallel to the surface.

SOME TYPES OF FORCES

1. WEIGHT (W)

The gravitational force on any object is called weight. It is always directed towards the centre of earth.

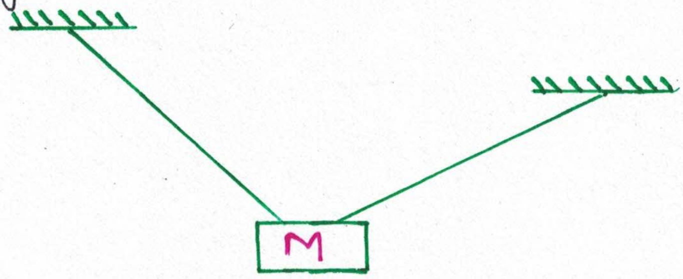


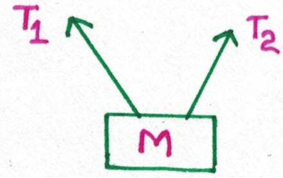
2. NORMAL REACTION (N)

whenever two surfaces are kept in contact, they exert a pushing force on each other due to repulsion (electromagnetic in origin) between the molecules. This pushing force acts perpendicular to the surfaces and is called normal reaction.

3. STRING TENSION (T)

Whenever an object is tied to a taut string, a pulling force acts on the object along the string (away from the object). This force is called string tension.



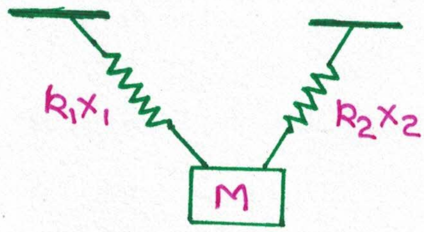


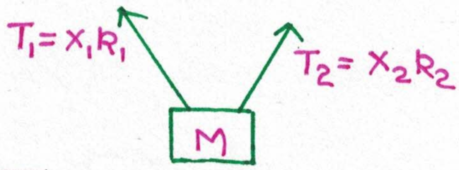
4. SPRING TENSION

Whenever an object is tied to a stretched spring, a pulling force acts on the object along the spring. This force is directly proportional to the elongation of the spring, and is known as spring tension.

Mathematically, $F = kx$

where x is the elongation in the spring & k is the constant which depends on the material and geometry of the spring, known as spring constant.

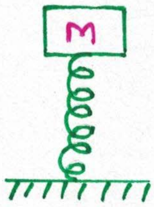


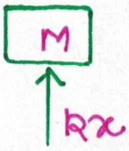


5. SPRING COMPRESSION

Whenever an object is attached to a spring in compression, a pushing force acts on the object which is proportional to the amount of compression in the spring.

Mathematically, $F = kx$





6. FRICTION

Whenever a surface has a tendency of relative motion with respect to another surface, a shearing force acts on it opposite to the tendency of relative motion.

NOTE: Apart from the forces listed before, we will come across some more contact and non-contact forces.

(a) CONTACT FORCES

1. Viscous Drag
2. Buoyant Force/Pressure Forces

(b) NON-CONTACT FORCES

1. Magnetic Force
2. Electrical Force

(c) NUCLEAR FORCES

NEWTON'S LAWS OF MOTION

FIRST LAW

A body continues to be in state of rest or uniform motion unless acted upon by an external unbalanced force, such a state is also called equilibrium.

Mathematically, $\sum \vec{F} = 0$ for equilibrium

SECOND LAW

Net force acting on a particle (or a body) is directly proportional to the rate of change of its momentum.

Mathematically,

$$\vec{F} \propto \frac{d\vec{P}}{dt}$$

$$\vec{F} = k\frac{d\vec{P}}{dt}$$

In S.I. units,

$$\vec{F} = \frac{d\vec{P}}{dt}$$

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$

$$\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

For constant mass systems $\frac{dm}{dt} = 0$

and our equation simplifies to,

$$\boxed{\vec{F} = m\vec{a}}$$

THIRD LAW

To every action, there is an equal and opposite reaction, and action and reaction always act on different bodies.

Strong form of Newton's Third law:

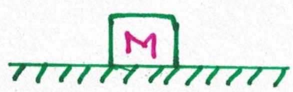
Action and reaction are along the same line.

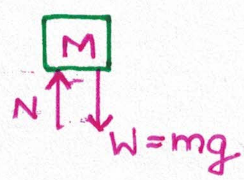
FREE BODY DIAGRAM

A diagram showing all the forces acting on a body is called a free body diagram.

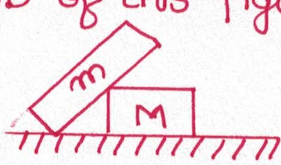
STEPS:

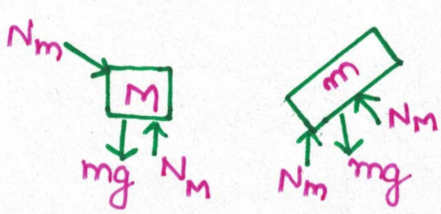
- Free the body (Remove all the contacting surfaces, ropes etc.) but remember where they were attached.
- Show the contact forces.
- Show the non-contact forces.



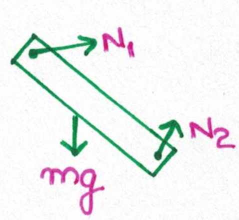


Q.) Draw FBD of this figure.





Q.) Draw FBD



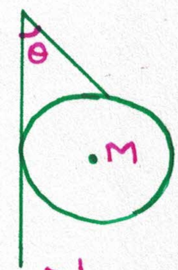
A system is said to be in equilibrium when it is not accelerated which automatically means that net force on it should be equal to zero i.e.

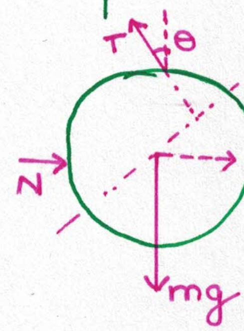
$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

Q1 Body is in equilibrium.
(i) Draw FBD of ball
(ii) Find the tension & the normal reaction.





$$Mg = T\cos\theta$$

$$N = T\sin\theta$$

$$\frac{N}{Mg} = \tan\theta$$

$$T = \frac{Mg}{\cos\theta}$$

Taking axis as T_H & T_L (T - Thread)

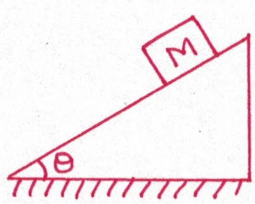
$$T = Mg\cos\theta + N\sin\theta$$

$$N\cos\theta = Mg\sin\theta$$

$$T = Mg\cos\theta + Mg\tan\theta . \sin\theta$$

$$N = Mg\tan\theta \quad , T = \frac{Mg}{\cos\theta}(\cos^2\theta + \sin\theta)$$

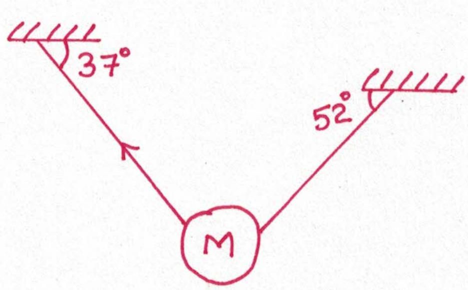
Q2

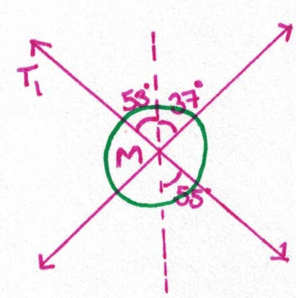


$$F = Mg\sin\theta$$

$$N = Mg\cos\theta$$

Q3





$$T_1 = Mg\cos 53° = \frac{3}{5}Mg$$

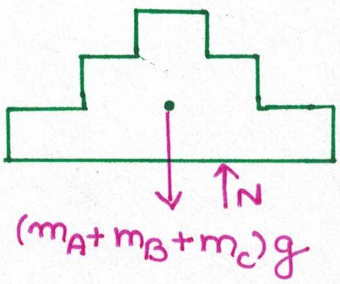
$$T_2 = Mg\sin 53° = \frac{4}{5}Mg$$

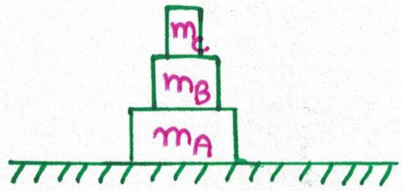
COMBINED FREE BODY DIAGRAM

When two or more objects do not move relative to each other then we can always treat them as a single object for the purpose of drawing FBD if it helps in making quicker calculations.

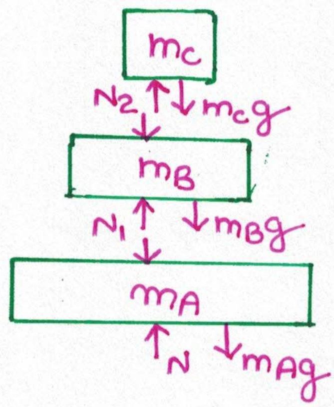
While drawing the combined FBD, the internal force b/w the objects are not shown.

For the shown system, find the normal reaction b/w ground and A. Compare the effort b/w drawing the combined FBD and individual FBD.





$N = (m_A+m_B+m_C)g$



$N_2 = m_Cg$

$N_1 = N_2 + m_Bg$

$N = N_1 + m_Ag$

on adding above

$N_1 + N_2 + (m_A+m_B+m_C)g$

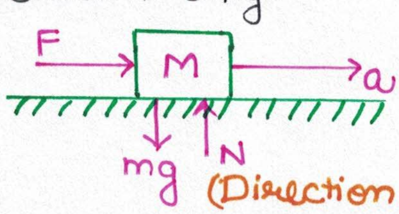
$= N + N_1 + N_2$

PROBLEMS INVOLVING ACCELERATION

In such problems, instead of balancing the forces in the direction of acceleration, we write the equation as

$$\Sigma F = ma$$

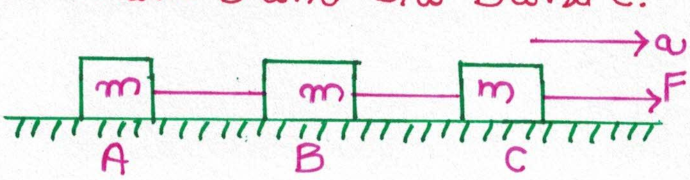
Note that even in problems of acceleration there may exist a direction equilibrium and in that direction we should have equations for equilibrium only.



(Direction of acceleration) $\Sigma F = ma$

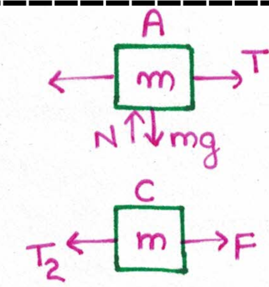
(Direction of equilibrium) $\Sigma F = 0$

Q. Find (i) acceleration of blocks
(ii) Tension b/w A and B and b/w B and C.



$$\vec{a} = \frac{\vec{F}}{3m}$$

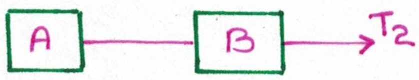
$$T_1 = m \times a$$
$$= \cancel{m} \times \frac{F}{3\cancel{m}} = \frac{F}{3}$$



$$F - T_2 = \cancel{m} \times \frac{F}{3\cancel{m}}$$

$$T_2 = F - \frac{F}{3} = \frac{2F}{3}$$

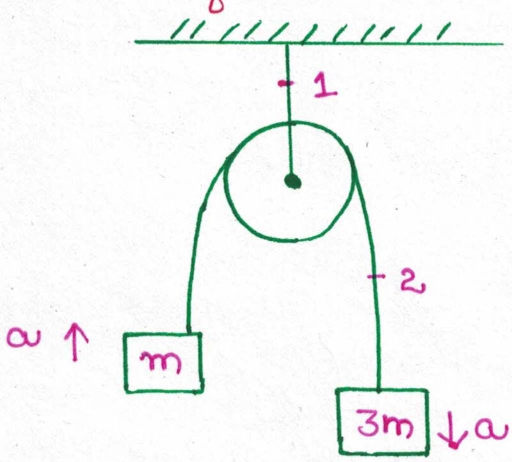
$$T_2 = 2\cancel{m} \times \frac{F}{3\cancel{m}} = \frac{2F}{3}$$

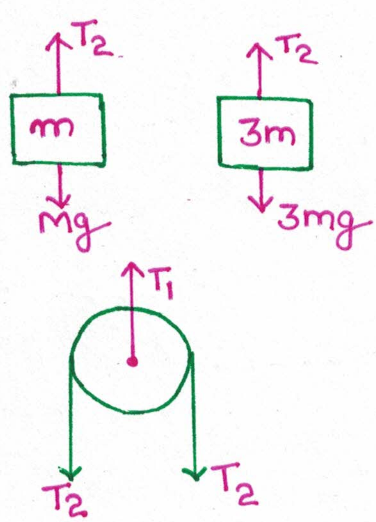


NOTE: 1.) The net force on a massless object is always zero even if it is accelerated.

2.) When a frictionless and a massless string passes over a pully, the tension is same on both sides of the string.

Q.) For the shown system the pully is massless & frictionless. Find
(i) T_1 & T_2
(ii) acceleration of block





$$3Mg - T_2 = 3ma \quad -①$$

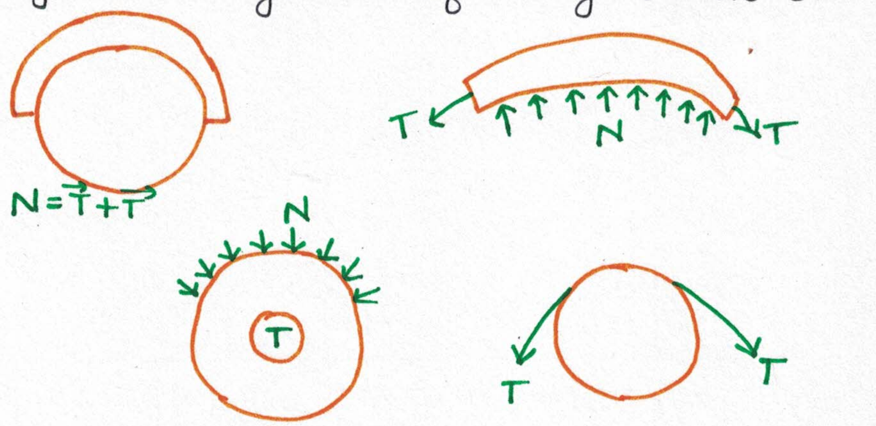
$$T_2 - Mg = Ma \quad -②$$

From both the equations.

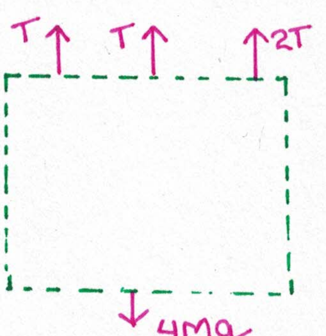
$$2\cancel{M}g = 4\cancel{m}a$$
$$a = \frac{g}{2}$$

$$2T_2 = T_1$$

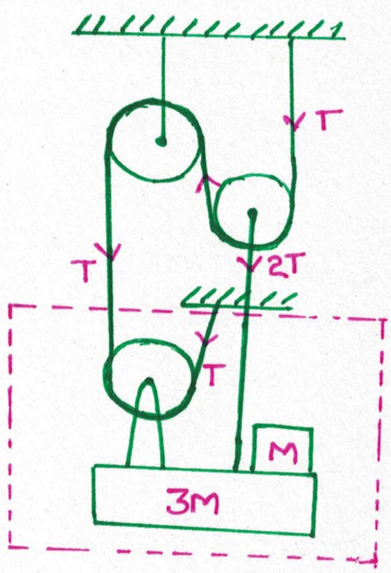
NOTE: whenever a taut string is passing over a pulley, the force exerted by it can be shown along the tangents of the string with magnitude of string tensions.



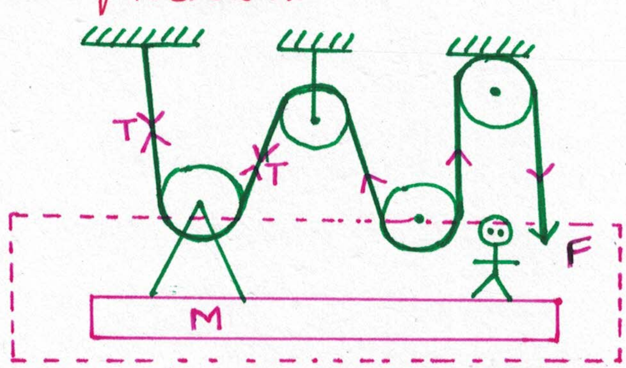
Q. Find the tension in the string



$$4T = 4Mg$$
$$T = Mg$$



Q. With what force the painter should pull the string so that system remains in equilibrium.

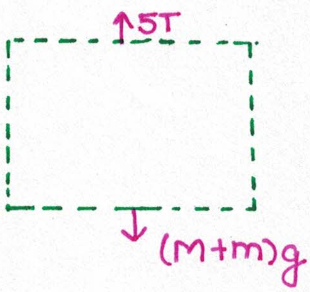


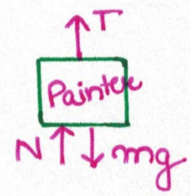
$$11\,Mg = 5T$$
$$T = \frac{11}{5}Mg$$

Q. Repeat the last problem if now the painter wants to accelerate upward with an acceleration g.

$$5T - 11Mg = 11Mg$$
$$T = \frac{22Mg}{5}$$

Q. Find the min. ratio of M:m so that the painter does not get lifted from the platform.

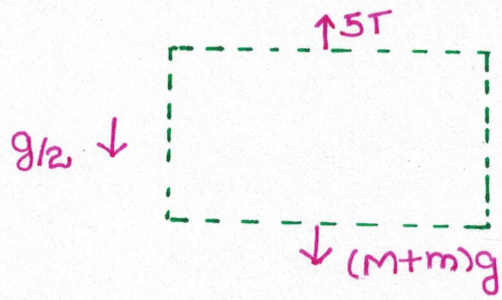


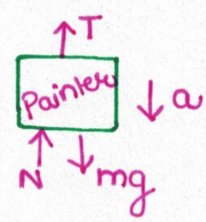


$$N + T = mg$$

$$5T = (M+m)g$$
$$5mg - 5N = Mg + mg$$
$$4mg = Mg + 5N$$
$$\frac{4mg - Mg}{5} = 0$$
$$4mg = Mg$$
$$\frac{M}{m} > \frac{4}{1}$$

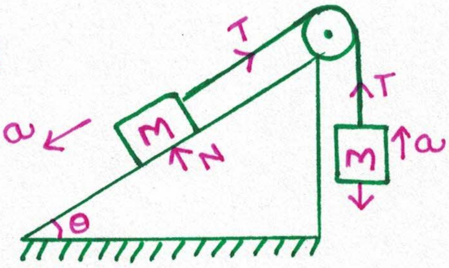
Q. Repeat the previous problem if we want the system going downward with an acceleration g/2.



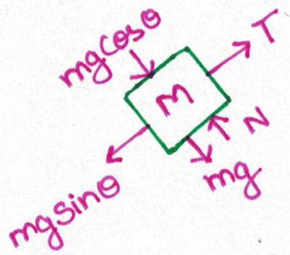


$$(M+m)g - 5T = (M+m)g/2$$
$$(M+m)\frac{g}{2} = 5T$$
$$T = \frac{(M+m)g}{10}$$
$$mg - T - N = \frac{mg}{2}$$
$$\frac{mg}{2} = T + N$$
$$N = \frac{mg}{2} - (M+m)\frac{g}{10} > 0$$
$$\frac{5mg - Mg - mg}{10} > 0$$

$$4mg - Mg > 0$$
$$4mg > Mg$$
$$\frac{4}{1} > \frac{M}{m}$$

Q.) 

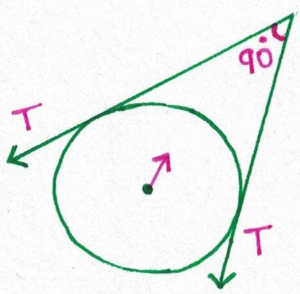
Find (i) Tension in string
(ii) acceleration
(iii) N
(iv) Pivot reaction on pulley



$$T - Mg = Ma$$
$$mg\sin\theta - T = ma$$
$$N = mg\cos\theta$$
$$mg\sin\theta - ma - mg = ma$$
$$2ma = mg(\sin\theta - 1)$$
$$a = \frac{\cancel{m}g}{2\cancel{m}}(\sin\theta - 1)$$
$$a = \frac{g}{2}(\sin\theta - 1)$$ {assume to opposite direction

$$\boxed{\text{Actual acceleration} = \frac{(1-\sin\theta)g}{2}}$$

$$T = ma + mg$$
$$T = m\left(g + \frac{g}{2}(\sin\theta - 1)\right)$$
$$T = mg\left(\frac{1+\sin\theta}{2}\right)$$

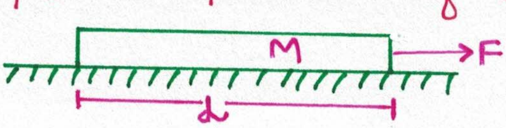


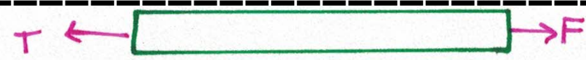
$$\text{Pivot force} = \sqrt{T^2 + T^2 + 2T^2\sin\theta}$$
$$= T\sqrt{2(1+\sin\theta)}$$

SECTIONAL FBD

Just as we can treat two different objects without a relative motion as a single object we can also treat a single object as an aggregate of various sections. When we draw FBD of a section of a body, it is a sectional FBD.

Q.) A uniform rope is pulled on a smooth surface. Find the tension in the rope as a function of distance x from front.





$$\frac{Mx}{L}$$

$$\vec{a} = \frac{F}{M} \quad \Rightarrow \quad F - T = \frac{M}{L}x \cdot a$$

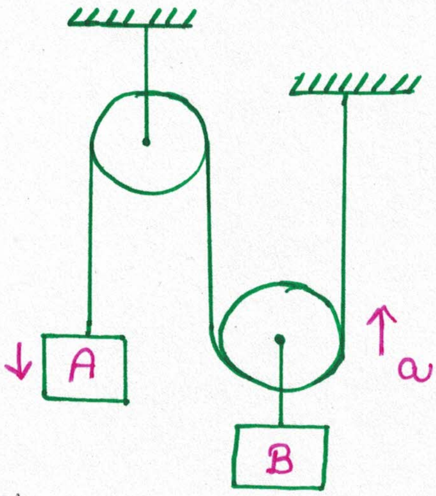
$$F - T = \frac{\cancel{M}}{L}x \times \frac{F}{\cancel{M}}$$

$$T = F - \frac{Fx}{L}$$

$$T = F\left[1 - \frac{x}{L}\right]$$

CONSTRAINT RELATIONS

In mechanical systems, due to the nature of connections the moti of some parts of a system automatically governs the motion of other parts. An equation representing such dependence is called the constraint equation.

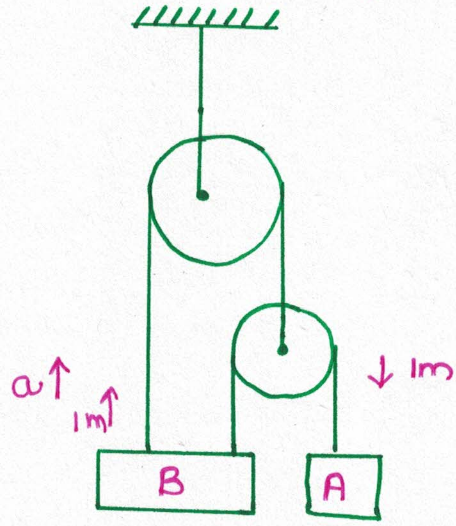


In the shown system the block B is moving with an upward acceleration and what is the acceleration of A?

by visual inspection method, $\vec{a}_A = 2a$

Q.) Repeat the previous question for the shown situation.

$\vec{a}_A = 3A$



In a system of massless and frictionless pulleys and strings, the total work done by the tension on all the blocks is equal to zero.

Q. Repeat the previous two problems by the tension work method.

(i) $2Ty\cos 0^\circ + Tx\cos 180^\circ = 0$

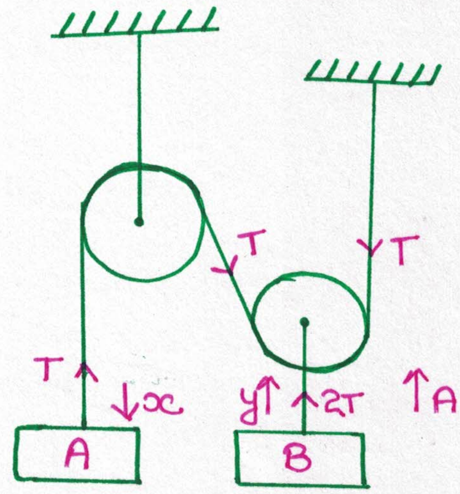
$$2Ty - Tx = 0$$

$$x = 2y$$

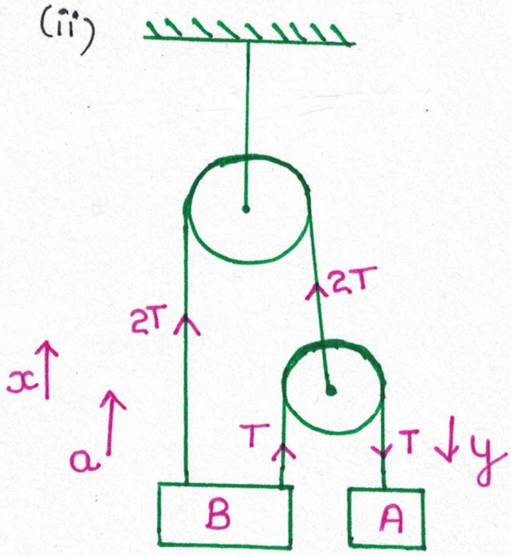
$$\frac{dx}{dt} = 2\frac{dy}{dt}$$

$$\frac{d^2x}{dt^2} = 2\frac{d^2y}{dt^2}$$

$$\vec{a}_x = 2a$$



(ii)



$$3Tx = Ty = 0$$

$$y = 3x$$

$$a_A = 3a$$

$$V_B = 7 m/s$$

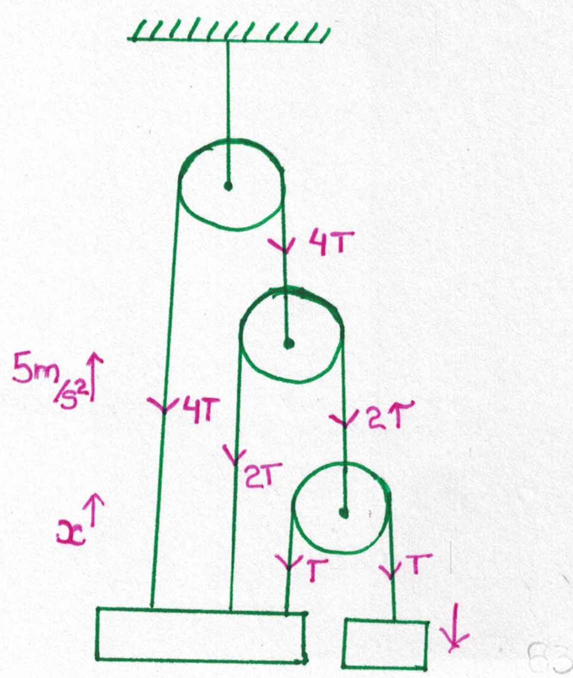
or

$$7Tx - Ty = 0$$

$$Ty = 7Tx$$

$$V_B = 7V_A$$

$$= 35 m/s$$



ROPE LENGTH METHOD

1:) Identify the number of moving bodies (if two bodies are moving together, they should be considered as a single body)

2:) Identify a study point on each moving body (usually end point of a rope or center of pulley).

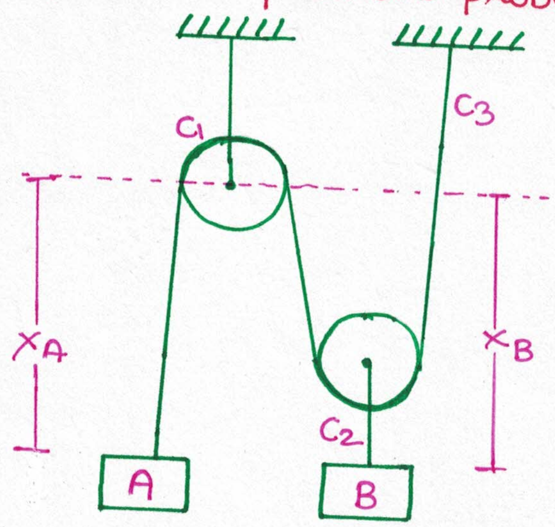
3:) Choose a reference line(s) and show the coordinate of each study point. (Generally, reference line is a line passing through center of a fixed pulley).

4:) Establish the constraints equations in terms of coordinates of study point using the fact that ropes have constant length.

5:) Differentiate these equations once or twice to get relation b/w velocity and acceleration.

Q:) Repeat the previous problems by using rope length method.

(i)



$$x_A + 2x_B + C_1 + C_2 + C_3 = \text{Const.}$$

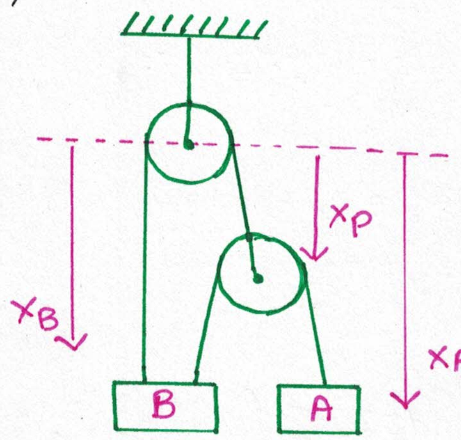
$$x_A + 2x_B = C$$

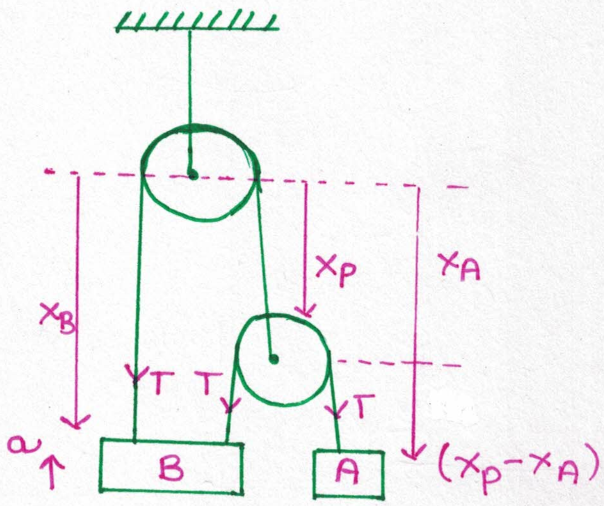
$$\vec{a}_A = -2\vec{a}_B$$

$$\vec{a}_A = -2(-\vec{a})$$

$$= 2a$$

(ii)





$$x_B + x_P = C \quad —①$$

$$x_B - x_P + x_A - x_P = C$$

$$x_B + x_A - 2x_P = C \quad —②$$

$$(a_B + a_P = 0)$$

$$a_B + a_A - 2a_P = 0$$

$$3a_B + a_P = 0$$

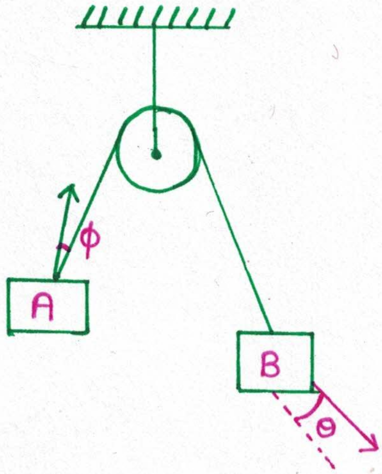
$$a_A = 3a_B$$

$$a_A = -3(-a)$$

$$= 3a$$

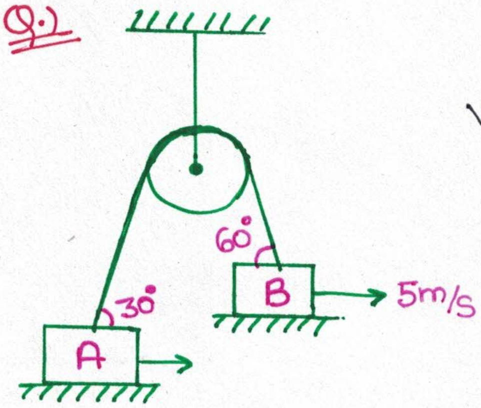
VELOCITY OF APPROACH METHOD

Whenever two objects are connected to a taut string, slung over a pulley then the components of their velocities along the string are equal.



$$V_A \cos\phi = V_B \cos\theta$$

Q.)



$$V_A \cos 30° = V_B \cos 60°$$

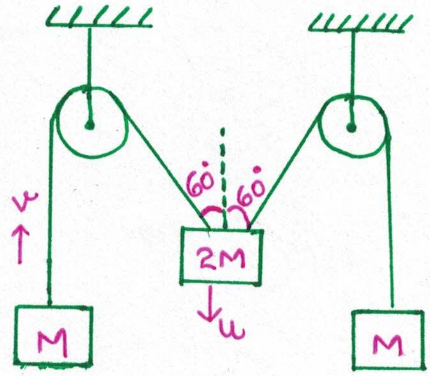
$$V_A = 5 \tan 30°$$

$$V_A = \frac{5}{\sqrt{3}} \, m/s$$

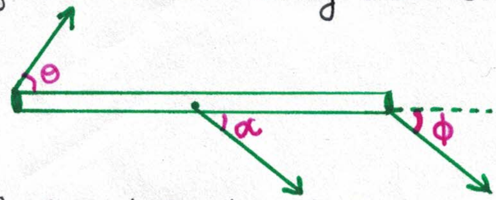
Q.)

$$u \cos 60° = v \cos 0°$$

$$v = \frac{u}{2}$$



NOTE: When two points lie on a rod/stick, their component of velocities along the stick is same.



$$V_A \cos\theta = V_B \cos\phi = V_C \cos\alpha$$

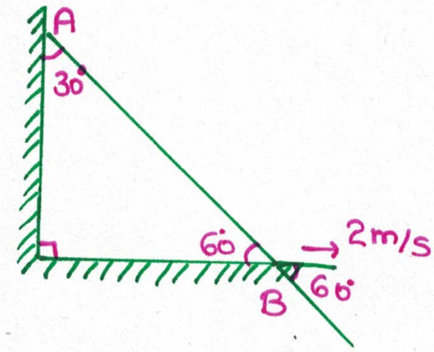
Q.)

$$V_B \cos 60^\circ - V_A \cos 30^\circ = 0 = V_{approach}$$

$$2\cos 60^\circ = V_A \cos 30^\circ$$

$$V_A = 2\tan 30^\circ$$

$$V_A = \frac{2}{\sqrt{3}}$$



Q.) Repeat the previous problem using the constraint between co-ordinates of A and B.

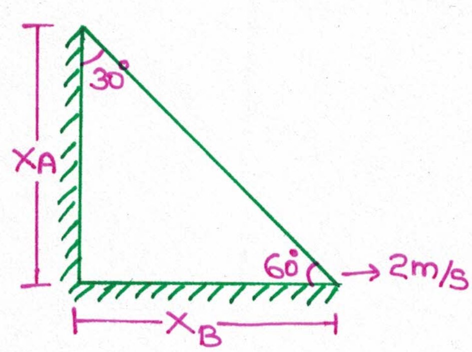
$$X_B^2 + X_A^2 = C$$

$$2X_B \cdot \frac{dX_B}{dt} + 2X_A \cdot \frac{dX_A}{dt} = 0$$

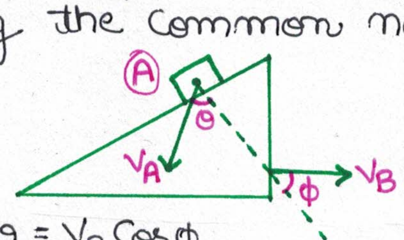
$$V_A = \frac{-X_B}{X_A} V_B$$

$$V_A = -\tan 30^\circ V_B \quad (V_B = +2)$$

$$V_A = \frac{-2}{\sqrt{3}}$$



NOTE: If two surfaces slip on each other, then their velocities along the common normal are equal.



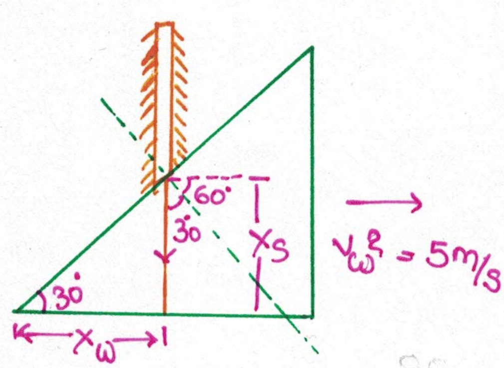
$$V_A \cos\theta = V_B \cos\phi$$

Q.)

$$V_S \cos 30^\circ = V_\omega \cos 60^\circ$$

$$V_S \frac{\sqrt{3}}{\cancel{2}} = 5 \times \frac{1}{\cancel{2}}$$

$$V_S = \frac{5}{\sqrt{3}} \text{ m/s}$$



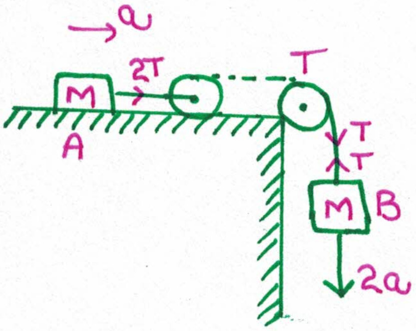
$$\frac{x_s}{x_\omega} = \tan 30^\circ$$

$$x_s = x_\omega \tan 30^\circ$$

$$V_s = V_\omega \tan 30^\circ$$

$$V_s = \frac{-5}{\sqrt{3}}$$

Q.) In the shown figure all the surfaces are frictionless. What will be acceleration of B if acceleration of A is a.



$$2T = Ma$$

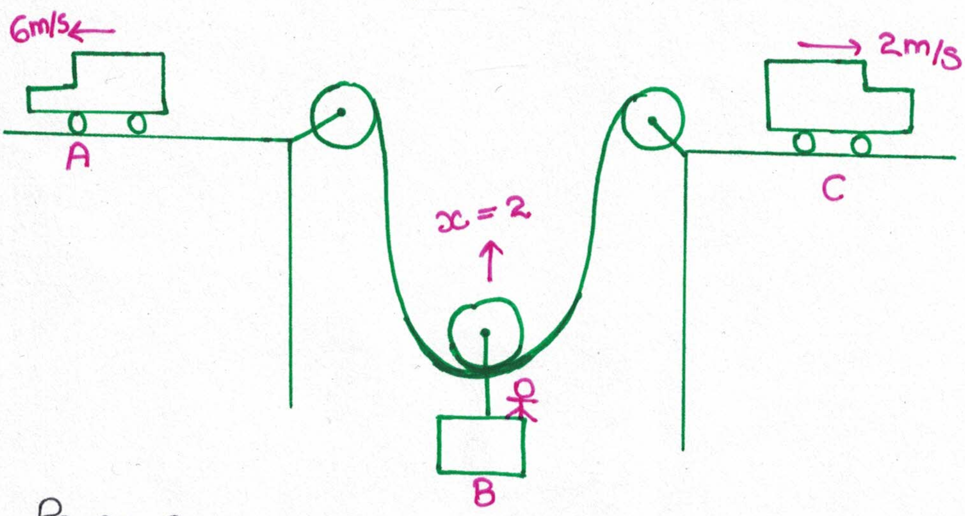
$$Mg - T = 2Ma$$

$$2Mg = 5Ma$$

$$a_A = \frac{2g}{5}$$

$$a_B = \frac{4g}{5}$$

Q.)



Pause C $\Rightarrow$ B will move with 3m/s

Pause A $\Rightarrow$ B will move with 1m/s

$\therefore$ B will move with 4m/s

PSEUDO FORCE

NON-INERTIAL FRAMES

Those frames of reference in which Newton's 2^{nd} law does not hold are called non-inertial frames or accelerated frames.

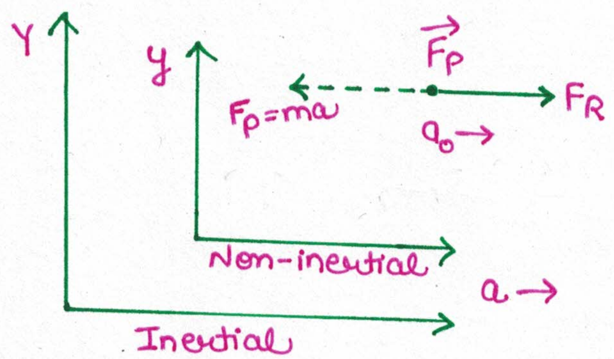
INERTIAL FRAMES

Those frames in which Newton's law hold good are called inertial frames or non-accelerated frames.

PSEUDO FORCE

A ficticious force which is introduced in the non-inertial frames to make the Newton's law valid is called Pseudo force.

CALCULATING PSEUDO FORCE (TRANSLATING FRAMES)



Consider a particle of mass 'm' being observed from reference frames 'Y' and 'y' (Y being inertial and y having an acceleration $\vec{a}$ with respect to Y. Further let $\vec{F_R}$ be the real force acting on the particle and let $\vec{F_P}$ vector be the required pseudo force to make the Newton's law valid in y. $\vec{a_o}$ is the acceleration of m with respect to Y.

$$\vec{F_R} = m\vec{a_o}$$

$$\vec{F_R} + \vec{F_P} = m(\vec{a_o} - \vec{a})$$

Subtracting bothe eq^n

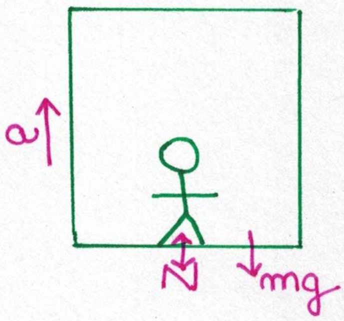
$$-\vec{F_P} = m\vec{a}$$

$$\boxed{\vec{F_P} = -m\vec{a}}$$ (opp. to direction of $\vec{a}$)

NOTE: 1) The direction of pseudo force is opposite to the acceleration of the observation frame.

2) The magnitude of pseudo force is mass of the observed objects times the acceleration of the observation frame.

Q.) A man is standing in an elevator which is moving upwards with an acceleration $\vec{a}$. Make equation for the Newton's law in the ground frame as well as lift frame and show that they are equivalent.

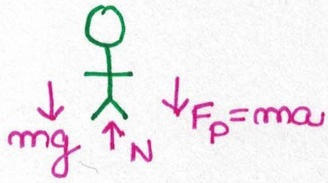


Ground frame

$$N - Mg = Ma$$
$$N = Mg + Ma$$

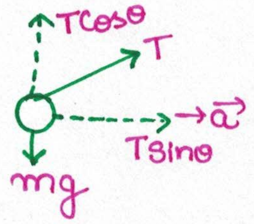
Lift frame

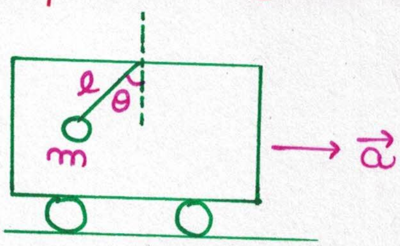
$$N = mg + \vec{F_P}$$
$$N = mg + ma$$



Q.) A rail road car is accelerated with an acceleration $\vec{a}$. What will be the angle made by a pendulam bob hanging inside the car. Analyze the problem from ground frame as well as car frame.

FBD

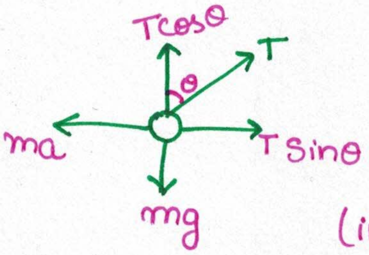




wrt ground frame

$$T\sin\theta = Ma$$
$$T\cos\theta = Mg$$
$$\tan\theta = \frac{a}{g}$$

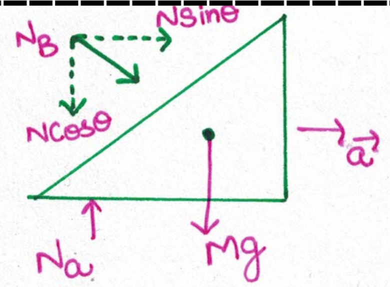
wrt car frame

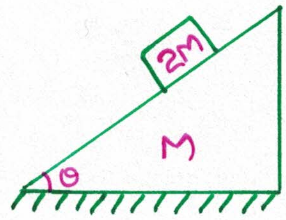


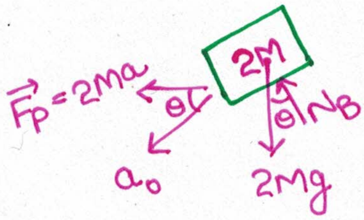
(in equilibrium)

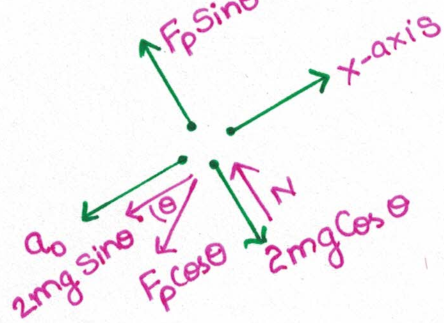
$$T\cos\theta = Mg$$
$$T\sin\theta = Ma$$
$$\tan\theta = \frac{a}{g}$$

Q.) In the shown system, all the surfaces are frictionless. Develop a system of equations to solve for acceleration of a wedge relative to ground and acceleration of the block relative to wedge. Can you also suggest a valid but useless equation for the current problem by using Newton's law.









For block :

$$2Ma\cos\theta + 2Mg\sin\theta = 2Ma_0 \quad \text{—(1)}$$

$$2mg\cos\theta = 2Ma\sin\theta + N \quad \text{—(2)} \quad \text{(equi.)}$$

For wedge

$$N\sin\theta = Ma \quad \text{—(3)}$$

$$N\cos\theta + Mg = Na \quad \text{—(4)} \quad \text{(valid but useless)}$$

Constraint equations with Moving Origin

Q.1 For the shown system develop a system of equations to solve for acceleration on the wedge and that of the block relative to wedge. Also make a valid but useless equation.

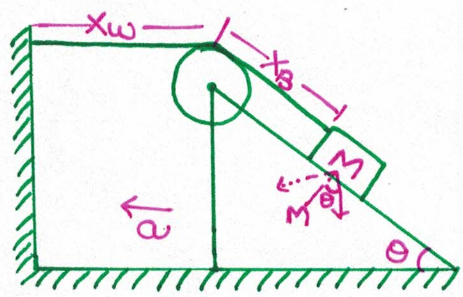
$$x_w + x_B = C$$

$$a_w + a_B = 0$$

$$a_B = -a_w$$

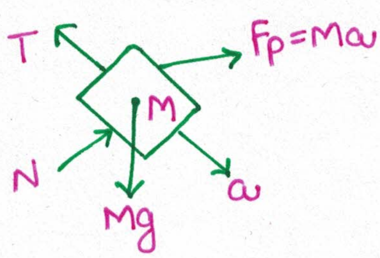
$$a_B = -(-a)$$

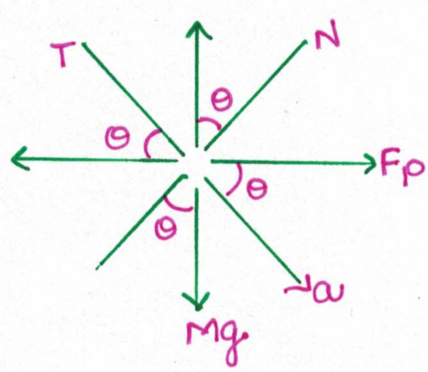
$$= a$$



(acceleration of block relative to wedge)

FBD of block



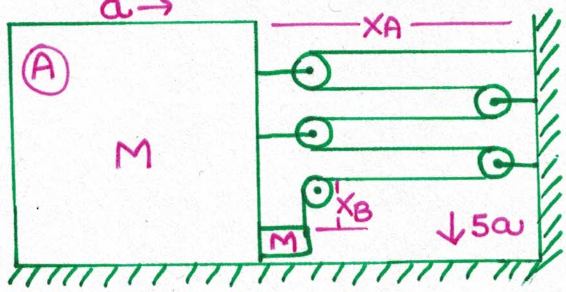


$$\vec{F_p} = Ma$$

$$N + F_p\sin\theta = Mg\cos\theta \quad \text{(equi.)}$$

$$Ma = F_p\cos\theta + Mg\sin\theta - T$$

Q.1 For the shown system, develop a system of equations to solve for the acceleration of (A) and (B)



$$5x_A + x_B = C$$

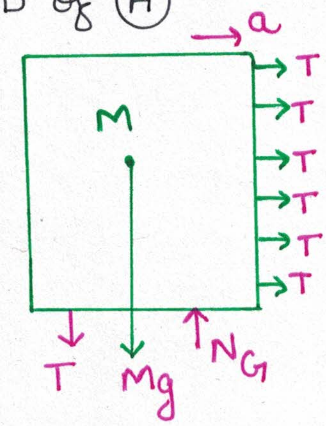
$$5a_A + a_B = C$$

$$a_B = -5a_A$$

$$a_B = -5(-a)$$

$$a_B = 5a$$

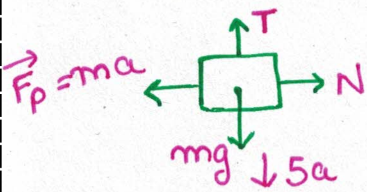
FBD of (A)



$$Ma = 5T - N$$

$$Mg = N_G - T \quad \text{(useless)}$$

FBD of (B)



$$Mg - T = 5Ma$$

$$N = ma$$

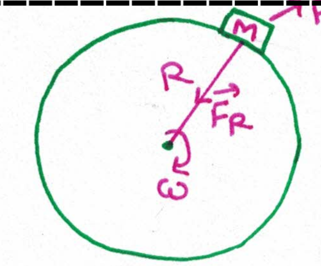
CALCULATING PSEUDO FORCE (ROTATING FRAMES)

Consider a block rotating on a smooth horizontal surface with an angular velocity ω and the radius of circle R. Let $\vec{F}_R$ be the force acting on the block, then from ground frame using Newton's 2nd law,

$$\vec{F}_P = m\omega^2 R(-\hat{c}) \quad \text{(Centrifugal Force)}$$

Now if we place an observer on the axis who constantly look towards the block. For such a rotating observer the accelerat? of the block appears to be 0. (To make Newton's law valid) If $\vec{F}_P$ be required pseudo force then,

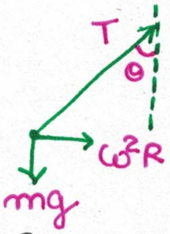
$$\vec{F_R} + \vec{F_P} = 0$$
$$\vec{F_P} = -\vec{F_R}$$
$$F_P = m\omega^2 R\,(-\hat{C})$$

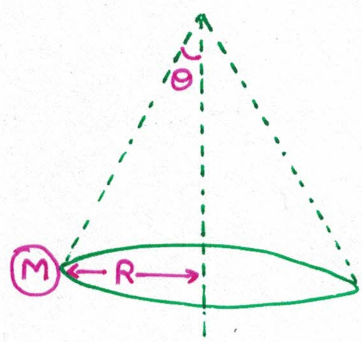


NOTE: 1.) This pseudo force acts away from the centre, and therefore known as **Centrifugal force.**

2.) The magnitude of the centrifugal force is the distance of the object from the frame times the mass of the observed object times the square of the angular velocity of the object.

Q. A conical pendulum rotates with an angular velocity ω in the circle of radius R. Find the angle θ that the string makes with the vertical. Analyse the problem from the ground frame as well as the rotating frame.

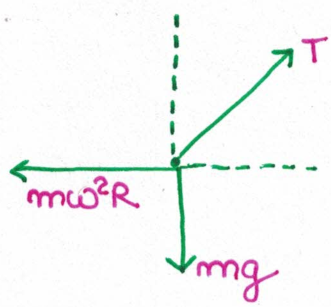




From ground frame,

$$T\sin\theta = m\omega^2 R$$
$$T\cos\theta = mg$$
$$\tan\theta = \frac{\omega^2 R}{g}$$

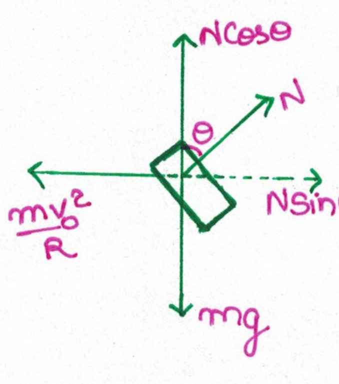
From rotating frame,

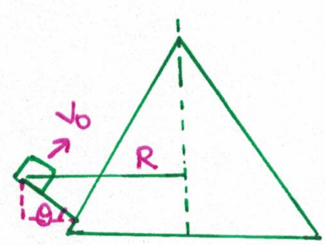


$$T\sin\theta = m\omega^2 R$$
$$T\cos\theta = mg$$
$$\tan\theta = \frac{\omega^2 R}{g}$$

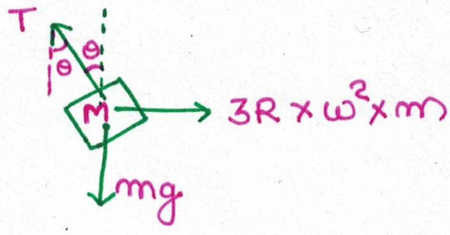
BANKING OF ROADS

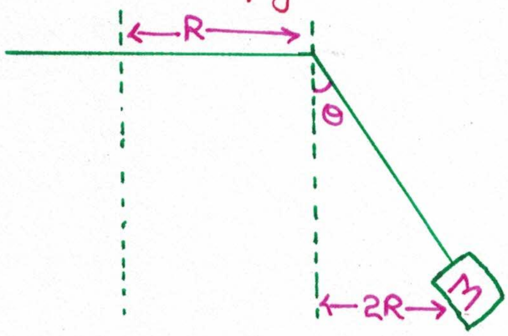
$$N\cos\theta = mg$$
$$N\sin\theta = \frac{m v_0^2}{R}$$
$$\tan\theta = \frac{v_0^2}{Rg}$$





Q.) Calculate the ω of the eagle ride shown in the figure.





$$T\cos\theta = mg$$

$$T\sin\theta = m\,3R\,\omega^2$$

$$\tan\theta = \frac{3R\omega^2}{g}$$

$$\omega = \sqrt{\frac{g\tan\theta}{3R}}$$